Name KEY Date _____

Exponential Functions

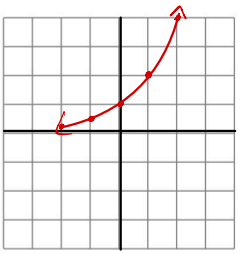
1) Complete each table for the exponential function. In each table, what do you notice about the values of x? What do you notice about the values of y?

x	$y = 16(2)^x$
0	16
1	32
2	64
3	128
4	256
5	512

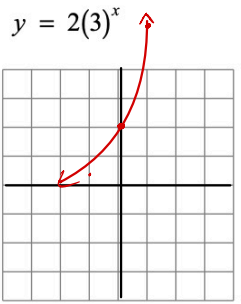
x	$y = 16(2)^x$
0	16
2	64
4	256
6	1024
8	4096
10	16384

5. Sketch the graph of each exponential function. Does each graph have the characteristics you described in Question 4? Explain your reasoning.

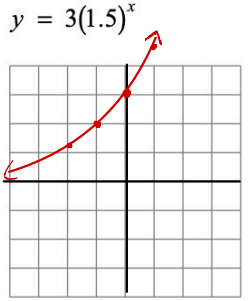
a. $y = 2^x$



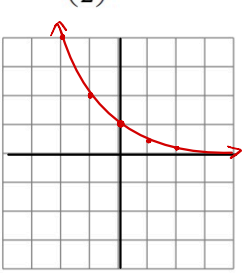
b. $y = 2(3)^x$



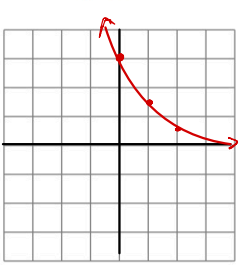
c. $y = 3(1.5)^x$



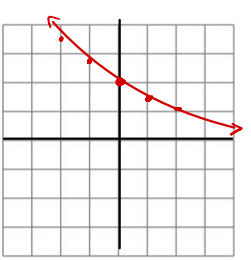
d. $y = \left(\frac{1}{2}\right)^x$



e. $y = 3\left(\frac{1}{2}\right)^x$



f. $y = 2\left(\frac{3}{4}\right)^x$



form $y = a(b)^x$

In Exercises 1–3, determine whether the equation represents an exponential function. Explain.

1. $y = -6^x$ Yes — variable in exponent ✓

2. $y = 5(1)^x$ No — Base is a 1 X

3. $y = 7x^3$ No — must be exponent not a base

In Exercises 4 and 5, determine whether the table represents an exponential function. Explain.

1.

x	y
1	8
2	4
3	2
4	1

×½, ×½, ×½

Yes

2.

x	y
1	3
2	7
3	11
4	15

+4, +4, +4

No (linear)

3.

x	y
−1	12
0	9
1	6
2	3

−3, −3, −3

No (linear)

4.

x	y
−1	0.125
0	0.5
1	2
2	8

×¼, ×¼, ×¼

Yes

Half-Life

1) A hospital prepared a 100-mg supply of technetium-99m, which has a half-life of 6 hours. Use the table below to help you understand how much of technetium-99m is left at the end of each 6-hour interval for 36 hours. Use this to help write an exponential function to find the amount of technetium-99m that remains after 75 hours.

The amount of technetium-99m is reduced by one half each 6 hours as shown in the table below. Fill in the missing information in the table and in the equation below.

Number of 6-hour Intervals	0	1	2	3	4	5	6
Number of Hours Elapsed	0	6	12	18	24	30	36
Amount of Technetium-99m (mg)	100	50	25	12.5	6.25	3.13	1.56

The amount of technetium-99m is an exponential function of the number of half-lives. The initial amount is 100 mg. The decay factor is ½. One half-life equals 6 hours.

Write an explicit equation if x = the number of 6-hour intervals. Y = $100(\frac{1}{2})^x$

If x = the number of hours elapsed, then the number of 6- hour intervals (of half-lives) = $\frac{1}{6}x = \frac{x}{6}$.

Equation: $y = 100\left(\frac{1}{2}\right)^{\frac{x}{6}}$

$y = 100(1/2)^{75/6}$

$y = 100(1/2)^{12.5}$

$y \approx 0.017$

After 75 hours, about 0.017 mg of technetium-99 remains.

2) Arsenic-74 is used to locate brain tumors. It has a half-life of 17.5 days. Write an exponential decay function for a 90-gram sample. Use the function to find the amount remaining after 6 days. (Hint: Make a table to help you understand the data.)

$y = 90\left(\frac{1}{2}\right)^{\frac{6}{17.5}} \approx 70.96\text{g}$

3) Phospohorus-32 is used to study a plant's use of fertilizer. It has a half-life of 14.3 days. Write the exponential decay function of a 50-mg sample. Find the amount of phosphorus-32 remaining after 84 days.

$y = 50\left(\frac{1}{2}\right)^{\frac{84}{14.3}} \approx 0.84\text{ mg}$

4) Iodine-131 is used to find leaks in water pipes. It has a half-life of 8.14 days. Write the exponential decay function for a 200-mg sample. Find the amount of iodine-131 remaining after 72 days.

$y = 200\left(\frac{1}{2}\right)^{\frac{72}{8.14}} \approx 0.43\text{ mg}$

5) Some radioactive ore which weighed 20 grams 200 years ago has been reduced to 12 grams today.

a. Use exponential regression on your calculator to write an exponential decay function in order to find the solution.

$y = 20(0.997)^x$

b. Based on your equation, what is the half-life of this radioactive ore?

$0.997^x = 0.5 \quad x = \frac{\ln(0.5)}{\ln(0.997)} \quad x \approx 231\text{years}$

c. Based on your half-life, write another exponential equation for the data in which the base of the exponent is ½.

d. How much will be left in 400 years?

$y = 20(0.997)^{400} \approx 6.02\text{g}$