

Name KEY

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

Date _____

1) Food poisoning is often caused by E. coli bacteria. To test for the presence of E. coli in a pot of beef stew, a biologist performs a bacteria count on a small sample of the stew kept at 25°C. She determines the count is 5 units per millilitre and the number will double every 40 minutes.

(a) Find the hourly growth factor a and find an exponential model $f(t) = Ca^t$ for the bacteria count in the beef stew.

1) Convert 40 min \rightarrow hour

$$40 \text{ min} \times \frac{1 \text{ hour}}{60 \text{ min}} = \frac{2}{3} \text{ hours}$$

2) Find hourly growth factor since it doubles every $\frac{2}{3}$ h

$$\hookrightarrow 2 = a^{2/3}$$

$$a = 2^{3/2}$$

$$a = \sqrt{8} \approx 2.828$$

2) Larry invested \$27,000 in a savings account that pays an annual interest rate of 1.8%. The savings account is set to compound quarterly (4 times per year). How much is in Larry's account after 5 years?

4 times per year

After 5 years = 29,536.74 \$

$$A = 27000 \left(1 + \frac{0.018}{4}\right)^{4 \cdot 5}$$

$$= 29536.74$$

3) Becky invested \$19,800 in a CD that pays an annual interest rate of 5.3%. The CD is set to compound daily. How much is in Becky's account after 9 years?

365 times per year

After 9 years = 31,901.32 \$

$$A = 19800 \left(1 + \frac{0.053}{365}\right)^{365 \cdot 9}$$

$$= 31901.32$$

4) Jake invests in an annuity with an annual fixed interest rate of 6.2%. The annuity compounds monthly. If after 10 years, the account balance is \$27,839.45 how much was the beginning investment?

12 times per year

Beginning balance = 15,000.00 \$

$$27,839.45 = P \left(1 + \frac{0.062}{12}\right)^{12 \cdot 10}$$

$$P = 15000.00$$

$$27,839.45 = P (1.005166667)^{120}$$

5) Jennifer is saving up for a house and wants a 20% down payment. She will invest a lump sum into a savings account for 5 years that pays 4.3% annual interest and compounds monthly. After some calculations, she figures her ideal house will cost \$140,000. How much should she put in the savings account?

$$140K - 100\%$$

$$28K - 20\%$$

$$28,000 = P \left(1 + \frac{0.043}{12}\right)^{12 \cdot 5}$$

$$P = \frac{28,000}{1.23918}$$

$$P \approx 22,595.45$$

12 times per year 22,595.45 \$

Instantaneous Rates of Change

1) The population of a culture of bacteria is given by $P(t) = 7t^2 + 4t + 1500$.

(a) Find the equation for the rate of change of the population after t hours.

$$P(t) = 14t + 4$$

$$P(t+h) - P(t)$$

$$1) P(t+h) = 7(t+h)^2 + 4(t+h) + 1500$$

$$(t+h)^2 = t^2 + 2th + h^2$$

$$= 7t^2 + 14th + 7h^2 + 4t + 4h + 1500$$

$$2) (7t^2 + 14th + 7h^2 + 4t + 4h + 1500) - (7t^2 + 4t + 1500)$$

$$\hookrightarrow 14th + 7h^2 + 4h$$

$$3) \hookrightarrow \frac{14th + 7h^2 + 4h}{h} = 14t + 7h + 4$$

$$4) \text{ let } h=0 \quad \leftarrow \quad 14t + 4$$

(b) What is the rate of change after 4 hours?

$$P(4) = 14(4) + 4$$

$$P(4) = 60$$

3. The population of a pride of lions over time (in years) is given by

$P(t) = 150(1 + 0.5t + 0.08t^2)$. What is the growth rate (in lions per year) when $t = 5$ years?

$$P(t) = 150(1 + 0.5t + 0.08t^2)$$

$$P(t) = 150 + 75t + 12t^2$$

$$1) P(t+h) = 150 + 75(t+h) + 12(t^2 + 2th + h^2)$$

$$= 150 + 75t + 75h + 12t^2 + 24th + 12h^2$$

$$2) 150 + 75t + 75h + 12t^2 + 24th + 12h^2 - (150 + 75t + 12t^2)$$

$$\hookrightarrow 75h + 24th + 12h^2$$

$$3) \frac{75h + 24th + 12h^2}{h} = 75 + 24t + 12h$$

$$4) \text{ let } h=0 = P'(t) = 75 + 24t$$

5) Plug in $t=5$

$$P'(5) = 75 + 24(5)$$

$$= 195$$

Compound Interest

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

Where,

P = principal amount (generated by *Compound Hustle*)
r = annual nominal interest rate (as a decimal)
n = number of times the interest is compounded per year
t = number of years

1) Evan opened a savings account and deposited $\$200.00$ as principal. The account earns 15% interest, compounded annually. What is the balance after 3 years?

Round your answer to the nearest cent.

\$ 304.18

$$\begin{aligned} A &= 200 \left(1 + \frac{0.15}{1} \right)^{1 \cdot 3} \\ &= 200 (1 + 0.15)^3 \\ &= 304.18 \end{aligned}$$

2) Deb puts $\$200.00$ into an account to use for school expenses. The account earns 9% interest, compounded annually. How much will be in the account after 4 years?

\$ 282.32

$$\begin{aligned} A &= 200 \left(1 + \frac{0.09}{1} \right)^{1 \cdot 4} \\ &= 200 (1 + 0.09)^4 \\ &= 282.32 \end{aligned}$$

3) Sandeep opened a savings account and deposited $\$100.00$. The account earns 14% interest, compounded annually. If he wants to use the money to buy a new bicycle in 3 years, how much will he be able to spend on the bike?

\$ 148.15

$$\begin{aligned} A &= 100 \left(1 + \frac{0.14}{1} \right)^{1 \cdot 3} = 148.15 \\ &= 100 (1 + 0.14)^3 \end{aligned}$$

4) Dylan and Irma deposit $\$500.00$ into a savings account which earns 8% interest compounded annually. They want to use the money in the account to go on a trip in 3 years. How much will they be able to spend?

\$ 629.86

$$\begin{aligned} A &= 500 \left(1 + \frac{0.08}{1} \right)^{1 \cdot 3} \\ &= 629.86 \$ \end{aligned}$$